



Cambridge International AS & A Level

CANDIDATE
NAME

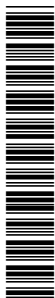
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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

3

- 2 Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

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- 3 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$ and $|z| \geq |z - 4i|$. [4]

4 The parametric equations of a curve are

$$x = \frac{\cos \theta}{2 - \sin \theta}, \quad y = \theta + 2 \cos \theta.$$

Show that $\frac{dy}{dx} = (2 - \sin \theta)^2$.

[5]

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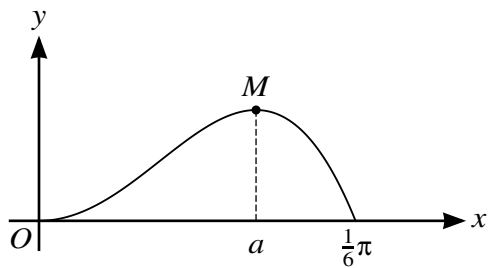
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The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leq x \leq \frac{1}{6}\pi$, and its maximum point M , where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3} \tan^{-1}\left(\frac{2}{3a}\right)$. [3]

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- 6 (a)** Express $3 \cos x + 2 \cos(x - 60^\circ)$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [4]

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(b) Hence solve the equation

$$3 \cos 2\theta + 2 \cos(2\theta - 60^\circ) = 2.5$$

for $0^\circ < \theta < 180^\circ$.

[4]

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7 (a) Use the substitution $u = \cos x$ to show that

$$\int_0^\pi \sin 2x e^{2\cos x} dx = \int_{-1}^1 2ue^{2u} du. \quad [4]$$

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(b) Hence find the exact value of $\int_0^\pi \sin 2x e^{2\cos x} dx$. [4]

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8 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$.

[8]

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9 The lines l and m have equations

$$l: \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$

$$m: \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O , the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

- (a) Given that l is perpendicular to m and that P lies on l , find the values of the constants a , b and c .
[4]

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- (b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the simplified exact values of r and θ . [3]

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Additional Page

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